

Probabilistic Uncertainty Modeling of Obstacle Motion for Robot Motion Planning*

Jun Miura and Yoshiaki Shirai

Dept. of Computer-Controlled Mechanical Systems,
Osaka University, Suita, Osaka 565-0871, Japan
phone: 06-6879-7332 facsimile: 06-6879-4185
jun@mech.eng.osaka-u.ac.jp

Abstract

This paper describes a method of modeling the motion uncertainty of moving obstacles and its application to mobile robot motion planning. The method explicitly considers three sources of uncertainty: path ambiguity, velocity uncertainty, and observation uncertainty. The uncertainty model represents the position of an obstacle at a certain time point by a probabilistic distribution over possible positions on each possible path of the moving obstacle. Using this model, the best robot motion is selected in a decision-theoretic way. By considering not the range but the distribution of the uncertainty, more efficient behaviors of the robot are realized.

Keywords: Uncertainty model of obstacle motion, Probabilistic uncertainty modeling, Robot motion planning, Mobile robot, Decision-theoretic planning.

1 Introduction

Motion planning is one of the fundamental functions of mobile robots. As mobile robots extend their application areas from factory to office or home, they have to cope with moving obstacles such as human or other robots. Therefore, mobile robot motion planning in dynamic environments has recently been studied extensively [1].

In the case where a robot cannot communicate with moving obstacles, the robot needs to predict the future motion of them. Most of past research can be classified, in terms of the knowledge of obstacle motion, into two categories. In one category, the obstacle motion is completely *unknown* and, therefore, a reactive motion selection is only reasonable way for a robot to cope with moving obstacles [2]; not the optimality but the safety of robot motion is an important issue there. In the other category, the obstacle motion is completely *known*; thus, an optimal motion can be generated by employing a *planning in space-time*[3]. In [4], for example, the robot predicts future motion of

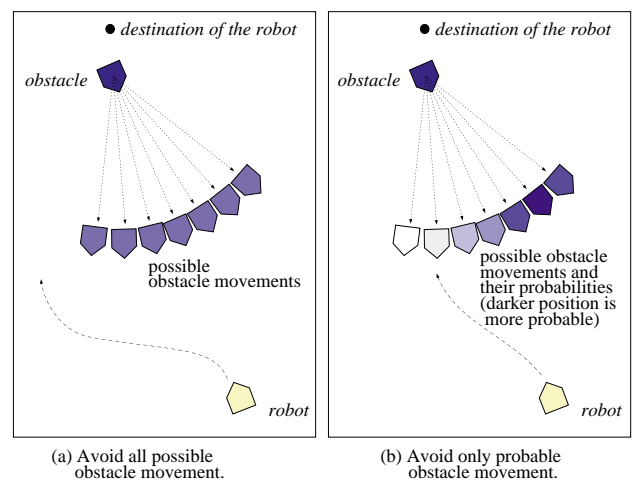


Fig. 1: An example case where considering the bias in the motion uncertainty is effective.

obstacles by assuming that they will continue to move at the current velocity, and plans the next best action in the space-time. Such prediction and planning are repeatedly performed. Fiorini and Shiller [5] proposed a local motion planning method using the concept of *velocity obstacles*.

In between these categories, several works consider the uncertainty in obstacle motion. Most of them, however, consider only the range of uncertainty (e.g., [6][7]); the robot generates a plan which is safe regardless of the actual obstacle motion. Such an uncertainty modeling may result in an inefficient robot motion if the positional distribution of an obstacle is not uniform within the range. Fig. 1 shows a typical case. In the figure, the robot is going to the destination, which is currently behind the obstacle. In case (a), the robot avoids all possible movements of the obstacle, while in case (b), it takes a path which is relatively short *and* has a small probability of meeting the obstacle. Although in some cases, motion (b) is worse than motion (a), such a situation is rare and, therefore, motion (b) is better in terms of the expected efficiency. As shown in this

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example, the probabilistic modeling of motion uncertainty is often effective in realizing efficient robot behaviors.

In order to model the motion uncertainty of obstacles, we hierarchically decompose it into two levels: the ambiguity in path selection and the motion uncertainty on each path. This is analogous to “path-velocity” decomposition [8] in motion planning.

First, let us consider the path ambiguity. In a usual environment where both static and moving obstacles exist, we can predict the motion of moving obstacles to some extent. They never move randomly; each has its own start and goal points and the path connecting them should be generated in some rational manner (e.g., by a minimum-length criterion). In a typical office environment, for example, flow of people is restricted by the placement of walls, doors, furniture, and so on. To enumerate possible paths of a moving obstacle, we use the *tangent graph* [9]. Using the given knowledge of static obstacles, a tangent graph is generated. Each path is represented as a set of consecutive segments on the graph. Skeletonization of free spaces in the environment, such as the one based on the Voronoi diagram (e.g., [10]), is also suitable for path candidate generation, especially for relatively narrow free spaces.

Concerning the motion uncertainty on a path, let us consider the following simple example. Suppose you are going to cross a street and a car is approaching you. You have to decide when to begin crossing the street (i.e., before or after the car passes). When the car is far away, predicting when the car will pass in front of you suffers from a large uncertainty because it is a prediction of a far future, and because the observation uncertainty is large for a far object. As time advances, however, the situation will be more certain and, at some time point, you will be able to make a decision with confidence. As seen from this example, for modeling motion uncertainty, we consider two sources of uncertainty: the velocity uncertainty of an obstacle and the observation uncertainty of the robot.

We apply the uncertainty model to robot motion planning. When the knowledge of obstacle motion is uncertain, the robot may be better to defer the decision on path selection until the situation is more certain. To realize such a behavior, we adopt a decision-theoretic planning [11] in motion planning.

2 Modeling Motion Uncertainty on a Path

2.1 Modeling Velocity Uncertainty

To model the velocity uncertainty, we assume the following on the motion of obstacles: each moving obstacle has the possible range of its velocity, represented as $[v_{min}, v_{max}]$; it changes the velocity at every *time step* ΔT ; the velocity for a time step is constant, and is randomly *and* independently selected within the above

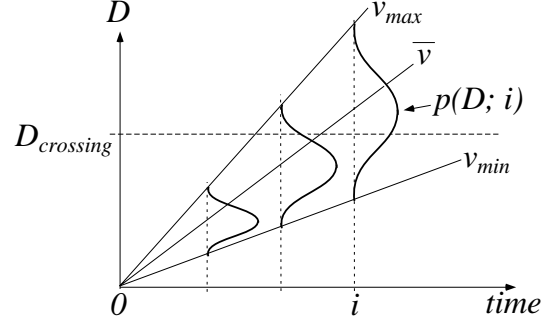


Fig. 2: Prediction of arrival time.

range¹.

Under these assumptions, we can predict the future position of a moving obstacle as follows. Let x_0 and σ_0^2 be the current position and the variance of an obstacle and v_k be the velocity at the k th time step. Then the position x_i after i steps is given by:

$$x_i = x_0 + \sum_{k=1}^i v_k \Delta T. \quad (1)$$

Since every v_k follows the same but independent uniform distribution within the above velocity range, the distribution of x_i can be *approximated* by a normal distribution (by *central limit theorem* [12]); the variance σ_{step}^2 of the motion added by one step is calculated as that of uniform distribution of width $v_{max} - v_{min}$, which is $(v_{max} - v_{min})^2/12$. The probability density function $p(x; i)$ of the obstacle being at x after moving for i steps is then given by

$$\begin{aligned} p(x; i) &= \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(x - \bar{x}_i)^2}{2\sigma_i^2}\right), \\ \bar{x}_i &= x_0 + i\bar{v}\Delta T, \\ \sigma_i^2 &= \sigma_0^2 + i\sigma_{step}^2, \end{aligned} \quad (2)$$

where $\bar{v} = (v_{max} + v_{min})/2$ is the mean velocity of the obstacle.

2.2 Predicting the Arrival Time of Moving Obstacle at a Crossing

Motion of an obstacle affects that of the robot near the crossings of their paths. Thus it is necessary for the robot to calculate the distribution of the arrival time of the obstacle at a crossing. Using the velocity uncertainty model described above, the distribution is calculated as follows (see Fig. 2).

In the figure, the vertical axis indicates the moving distance of the obstacle from the current position; the horizontal axis indicates the time (or time step). $D_{crossing}$ is

¹ More constraining knowledge could be used depending on the actual environment and the problem settings.

the distance to a specific crossing on the path. Since the positional distribution of the obstacle at some time point is calculated by eq. (2), the probability $P(i)$ of the obstacle reaching the crossing at the i th time step can be approximated by:

$$P(i) = \alpha p(D_{crossing}; i), \quad (3)$$

where α is a normalization constant.

2.3 Modeling Observation Uncertainty

The other source of uncertainty in predicting obstacle motion on a path is the observation uncertainty. We suppose a vision-based mobile robot which uses stereo vision to detect obstacles and to measure their position and velocity. We use the uncertainty model of stereo vision that we have previously developed [13]. The model uses a normal distribution to represent the positional uncertainty of an object due to vision uncertainty.

Every time the robot measures the position of an obstacle, the position data is statistically integrated with the previous data to reduce the uncertainty. Here we explain how to estimate the positional uncertainty of an obstacle after observation at the next step.

Let $N(\mu_0, \sigma_0^2)$ be the *predicted* distribution of obstacle position x before the next observation; this distribution is calculated from the current distribution and the predicted motion uncertainty added by the next step. Let x_{obs} be the position measured by the next observation. Assuming that the variance σ_{obs}^2 of x_{obs} is constant regardless of the true value of x , x_{obs} follows $N(\mu_0, \sigma_0^2 + \sigma_{obs}^2)$. From these values, mean μ_1 and variance σ_1^2 of the distribution after integrating the next observed data are estimated as follows; σ_1^2 is given by $\sigma_0^2 \sigma_{obs}^2 / (\sigma_0^2 + \sigma_{obs}^2)$. μ_1 cannot be predicted deterministically; instead, its *distribution* can be estimated as $N(\mu_0, \sigma_0^2 / (\sigma_0^2 + \sigma_{obs}^2))$. Refer to [13] for the details. We use this distribution of μ_1 to enumerate a set of possible states after the next step.

2.4 Gradual Reduction of Prediction Uncertainty

By combining the velocity and the observation uncertainty, we can model the gradual reduction of prediction uncertainty, which is described by an example in Sec. 1. Fig. 3 shows the current probability distribution of an obstacle arriving at a crossing, and a set of *predicted* probability distributions (weighted with their occurrence probabilities) to be obtained after one time step passes and the new observation result is integrated². The set of distributions covers all possible situation which is represented by the current distribution. Only one of which, however, will

² Note that the velocity range of the obstacle is discretized with some granularity for a computational purpose.

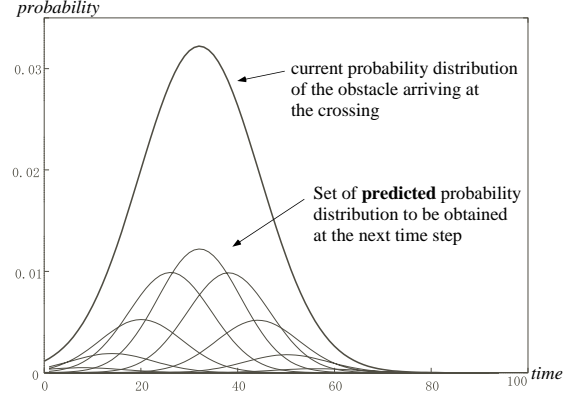


Fig. 3: Reduction of prediction uncertainty.

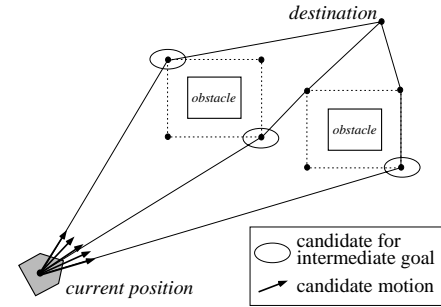


Fig. 4: Generating candidate motions.

actually occur. The variance of each distribution in the set is smaller than that of the current one, that is, the situation will become more certain.

3 Motion Planning for Fixed Path Moving Obstacle

3.1 Planning the Next Motion

The robot basically follows a path on the tangent graph to minimize the moving distance to the destination as long as there is no influence from moving obstacles.

If the robot has to consider avoidance of collision with them, the robot selects a certain number of nodes as the candidates of an intermediate goal and enumerates a set of candidate motions which cover the directions to the candidate nodes (see Fig. 4). After each time step, the robot observes obstacles, estimates their positional uncertainty, and performs one-step look-ahead search for the next motion which minimizes the expected time to the destination. If a node is known to be far superior to the others, the commitment is made to the node.

The detailed planning algorithm is as follows (see Fig. 5). For each motion i ($i = 1, \dots, N$), the robot first predicts the set of possible states $\{S_{ij} | j = 1, \dots, M\}$ with their probability P_{ij} , which are to be obtained after the

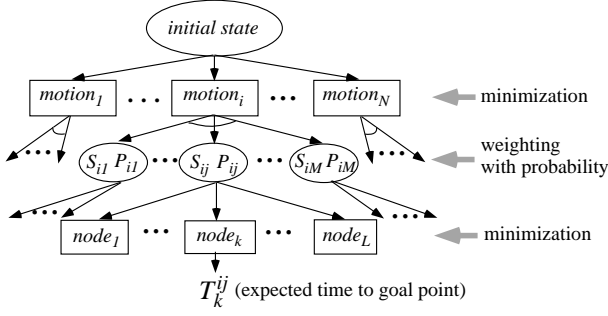


Fig. 5: Selecting the next motion.

motion. Each state holds a set of the positional distributions of moving obstacles, which is calculated as described in the previous section. Then for each state S_{ij} , the robot calculates the expected time T_k^{ij} of reaching the destination when selecting candidate node k ($k = 1, \dots, L$)³ and selects the best (minimum-time) candidate node k_{ij}^* as:

$$k_{ij}^* = \arg \min_{k=1}^L T_k^{ij}. \quad (4)$$

Then the expected time T_i of reaching the destination when taking candidate motion i is given by

$$T_i = \sum_{j=1}^M P_{ij} T_{k_{ij}^*}^{ij}. \quad (5)$$

Finally the best motion i^* is selected as:

$$i^* = \arg \min_{i=1}^N T_i. \quad (6)$$

3.2 Calculating Expected Time to Destination

Collision Avoidance by Stopping We set a safety distance L_{safe} and controls the robot so as not to enter within L_{safe} from any obstacles. Basically the robot moves at a constant speed on the shortest path. If the path of the robot and that of an obstacle intersect, and if the robot knows the obstacle will come to the distance less than L_{safe} , the robot stops before the intersection point (crossing) and waits for the obstacle to pass by⁴.

Calculating Waiting Time The period during which the robot has to wait is calculated as follows. Let us consider Fig. 6. The two paths intersect at p_c with angle θ . The robot waits at point p_0 whose distance to the path of the obstacle is L_{safe} . Let t_0 be the time at which the robot reaches p_0 . To calculate the waiting period, we first calculate two distances, D_{safe}^{in} and D_{safe}^{out} . D_{safe}^{in} indicates

the distance of the obstacle to the crossing p_c such that the robot can pass the crossing before the obstacle if the obstacle is further than D_{safe}^{in} at t_0 (see Fig. 6(a)). D_{safe}^{out} is the distance from the crossing such that the robot can pass the crossing after the obstacle if the obstacle is further than D_{safe}^{out} at t_0 (see Fig. 6(b)). Assuming that near the crossing, the obstacle and the robot moves at constant speed v_o and v_r respectively, these two distances are given by:

$$D_{safe}^{in} = \frac{L_{safe}}{\sin \theta} \left\{ \sqrt{\frac{v_r^2 + v_o^2 - v_r v_o \cos \theta}{v_r^2}} + \frac{v_o}{v_r} \right\}, \quad (7)$$

$$D_{safe}^{out} = \frac{L_{safe}}{\sin \theta} \left\{ \sqrt{\frac{v_r^2 + v_o^2 - v_r v_o \cos \theta}{v_r^2}} - \frac{v_o}{v_r} \right\}. \quad (8)$$

If the obstacle is within the range $[p_c - D_{safe}^{in}, p_c + D_{safe}^{out}]$ at t_0 , the robot has to wait for the obstacle exiting from the range. From this condition, we can obtain the following:

1. If the time of the obstacle arriving at the crossing is within the range $[t_0 - D_{safe}^{out}/v_o, t_0 + D_{safe}^{in}/v_o]$, the robot has to wait.
2. For arrival time t within the range, the robot has to wait for the duration of $t - (t_0 - D_{safe}^{out}/v_o)$ (see Fig. 7); this is explained as follows. If the obstacle arrives at p_c at t , we know that it was at the distance of $v_o(t - t_0)$ to p_c at time t_0 . Thus the robot has to wait while the obstacle moves by the distance $D_{safe}^{out} + v_o(t - t_0)$. Dividing this distance by v_o leads to the above waiting time.

Expected Time to Destination From the above result and the probability distribution $P(i)$ of the obstacle arriving at the crossing (see eq. (3)), we can calculate the expected time of the robot reaching the destination on a certain path. The expectation of the extra time needed for waiting, T_{wait} , is calculated by:

$$T_{wait} = \sum_{t \in [t_{min}, t_{max}]} P(t)(t - (t_0 - D_{safe}^{out}/v_o)), \quad (9)$$

$$t_{min} = t_0 - D_{safe}^{out}/v_o,$$

$$t_{max} = t_0 + D_{safe}^{in}/v_o.$$

The expected time to the destination is then calculated as the sum of T_{wait} and the time needed in the case where the robot encounters no obstacles.

The motion uncertainty of the robot is not considered in the above discussion. However, if the uncertainty is represented by a probabilistic distribution, it can easily be incorporated by further calculating the expectation over the possible range of robot uncertainty.

³ The next subsection will explain how to calculate the expected time.

⁴ Other avoidance methods such as potential methods can be used instead, if the corresponding procedure is available to calculate the expected time to reach the destination.

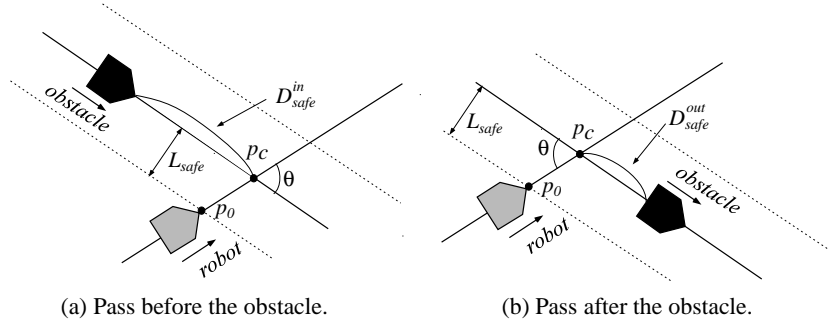


Fig. 6: Conditions for collision avoidance by stopping.

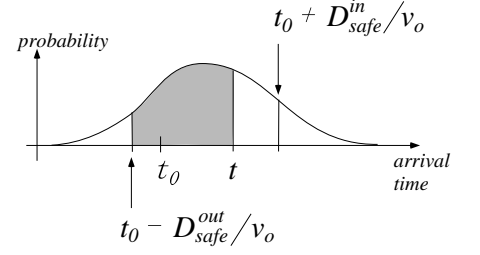


Fig. 7: Expected time to wait.

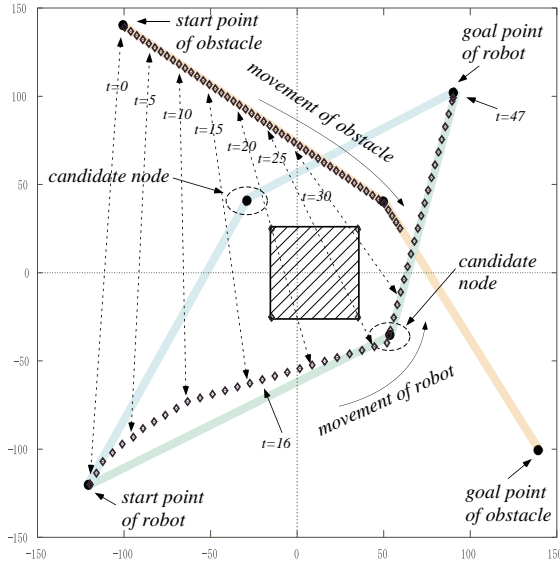


Fig. 8: A simulation result.

3.3 Simulation Results

Fig. 8 shows a simulation result. There are a static obstacle and a moving obstacle in the environment; the robot considers two paths, among which the left one is shorter. The figure shows the movements of the robot and the obstacle until the robot reached the goal point. Since the left path is shorter and the left and the right path are comparable due to a large uncertainty of obstacle motion, the robot started toward the left path; as the situation became more certain, the evaluation of the right path went up, while that of the left one fell down. So the robot gradually shifted its direction towards the right path and, at time $t = 16$, it committed to the right path and followed it to the goal point. The parameters used in this simulation are: obstacle velocity range is $[4.2 \pm 1.0][cm/s]$; the robot velocity is $7.5[cm/s]$; the variance of the uncertainty in measuring distance is $6.25e - 7 \cdot d^4$ (d is the distance to the obstacle); the length of the left and the right path are $317.0[cm]$ and $332.0[cm]$, respectively. The number of candidate motions

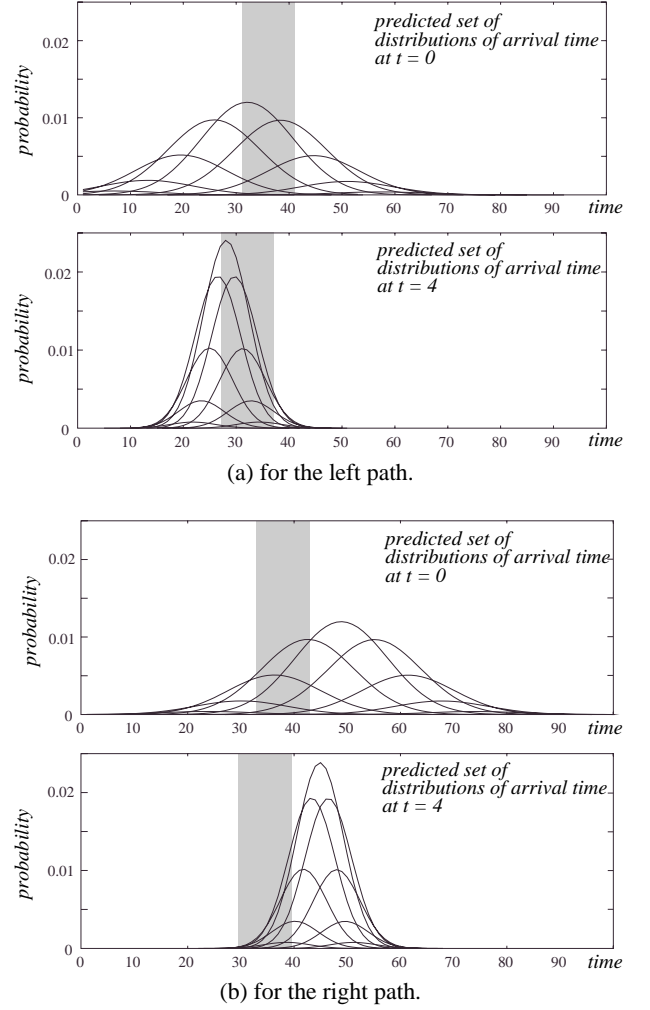


Fig. 9: Predicted distributions of the obstacle's arrival time at the crossing and the time range within which the robot has to wait.

and that of predicted states are 5 and 9, respectively.

In Fig. 9, each set of distributions indicates the predicted distributions of the arrival time of the obstacle at the

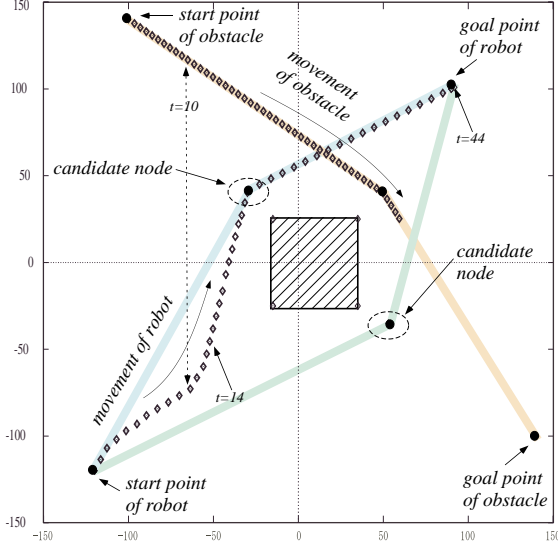


Fig. 10: Another result when the obstacle changes speed.

crossing at times $t = 1$ and $t = 4$ for the left and the right path, respectively. A shaded area in the figure indicates the time range $[t_0 - D_{safe}^{out}/v_o, t_0 + D_{safe}^{in}/v_o]$ of the obstacle's arrival, for which the robot has to wait (see Sec. 3.2). From the figures, we can see that as the situation becomes more certain, the evaluation of the right path is more improved than that of the left one. This evaluation improvement of the right path made the robot shift the moving direction to the right in Fig. 8.

Fig. 10 shows another result for the case where the obstacle increased the speed by about 5% at time $t = 10$. In response to this speed change, the robot changed the direction to the left; however it had not committed to the left yet due to the uncertainty of obstacle motion. At time $t = 14$, the robot finally committed to the left path and followed it. If the obstacle increases the speed more rapidly, the robot will make a commitment immediately.

4 Modeling Path Ambiguity

This section deals with the case where there are multiple possible paths for a moving obstacle. Suppose an obstacle is moving on a segment towards a node (*branching point*) to which n possible paths are connected. Let P_{path_j} ($j = 1, \dots, n$) is the probability of taking the j th path; if the robot does not have any prior knowledge of obstacle motion, $P_{path_j} = 1/n$.

To path ambiguity cases, we can also apply the motion selection method described in Sec. 3.1 with a little modification. For each possible path, we can calculate the expected time T_{ij} when taking candidate motion i using eq.

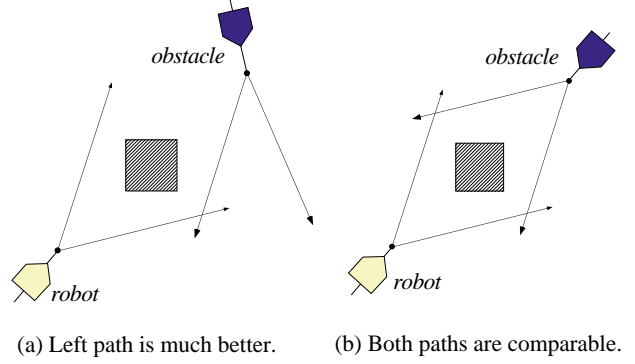


Fig. 11: Two cases of path ambiguity of obstacle.

(5). Then the expected time T_i of motion i is calculated by

$$T_i = \sum_{j=1}^n P_{path_j} T_{ij}. \quad (10)$$

Finally the best action is selected using eq. (6). But this method may not be appropriate in some cases.

In the case of Fig. 11(a), for example, the action towards the left path will be selected because this motion is apparently better than the other regardless of the actual path of the obstacle. In the case of Fig. 11(b), however, the above selection method is not effective; that is, each motion of the robot is good for one obstacle path but bad for the other. Since the motion is selected by one-step look-ahead search using the expected time to the destination, and since any motion is not far superior to the others, the motion towards directly one of the candidate nodes may be selected. But such a motion is very inefficient when the obstacle happens to take the path on the same side.

A reasonable strategy is, thus, to move towards some midpoint of the paths and to defer the commitment to a path until the obstacle takes one of the possible paths. The problem is, however, that it cannot be deterministically determined when the obstacle comes to the branching point. To cope with this problem and to calculate the expected time to the destination, we calculate and use the probabilistic distribution of the obstacle reaching the point, just as in the case of the obstacle reaching a crossing (see Sec. 2.2).

Let us examine the case where there is one moving obstacle and where there are two possible paths (*left* or *right*) from the next branching point for the obstacle. Let P_{left} and P_{right} be the prior knowledge of the probability of the obstacle taking the left or the right path, respectively. Also let $P(i)$ be the probability that the obstacle arrives at the branching point at time step i (see eq. (3)) and t_{min} and t_{max} be the earliest and the latest time of arrival, respectively. Assuming that the path of the obstacle is immediately determined when the obstacle passes the branching point, the probabilities P_{left}^i and P_{right}^i of the robot detecting the obstacle on the left and the right path, respectively,

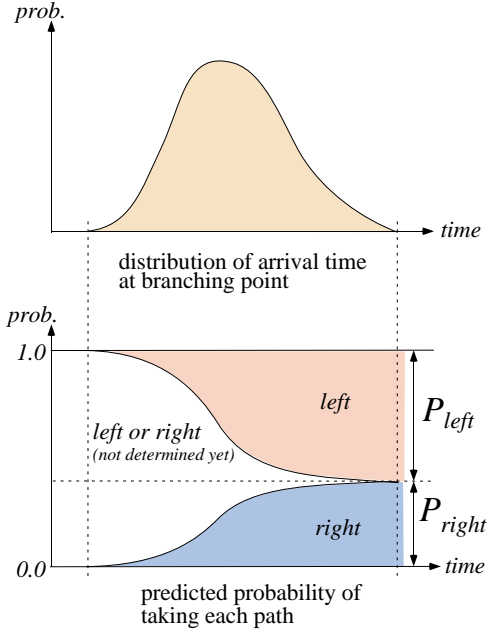


Fig. 12: Predicting probability of an obstacle taking each path.

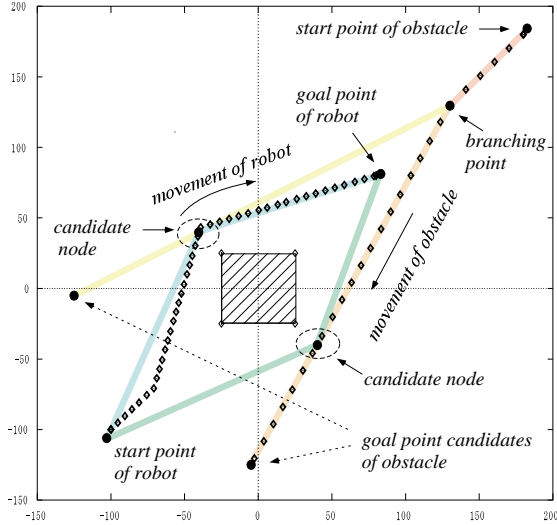


Fig. 13: Simulation result for the case of path ambiguity of a moving obstacle.

at time step i are calculated as:

$$P_{left}^i = P_{left}P(i), \quad P_{right}^i = P_{right}P(i). \quad (11)$$

Fig. 12 illustrates how the situation gradually becomes certain as the time advances in this model.

The expected time of the robot reaching the destination is calculated as follows. Let T_{left}^i and T_{right}^i be the time

of the robot reaching the destination when taking the best action against the left and the right path of the obstacle, respectively, after moving for the duration of i time steps without making commitment to any path. Then the expected time T of this strategy is given by

$$T = t_{min}\Delta T + \sum_{i=t_{min}}^{t_{max}} (P_{left}^i T_{left}^i + P_{right}^i T_{right}^i). \quad (12)$$

This expected time is compared with that of the action selected by the previous, one-step look-ahead strategy (see Sec. 3.1) and the better action is selected. This formulation can be extended to a general, n -paths case.

Fig. 13 shows a simulation result for the case where the robot considers the path ambiguity of a moving obstacle. Until time $t = 6$, the robot moved toward the center of the two candidate nodes and at time $t = 7$, it changed the direction to the left path because it detected the obstacle took the right path at that time. The parameters used in this simulation are: obstacle velocity range is $[15.3 \pm 1.0][cm/s]$; the robot velocity is $7.5[cm/s]$; the other parameters are the same as in the previous simulation shown in Fig. 8.

The above strategy of simply deferring the decision while moving at a constant speed may be inappropriate to some cases. For example, if the static obstacle in Fig. 13 is nearer to the start point of the robot, the robot has less time before selecting one of the paths. In this case, the robot could increase the time for collecting more information by slowing down the speed. At the same time, however, the robot would have to considering the tradeoff between the merit of path selection based on more certain information with the demerit of slowing down.

5 Conclusions and Discussion

We have proposed a method of modeling the motion uncertainty of moving obstacles to be used for robot motion planning. We consider three sources of the uncertainty, path ambiguity, velocity uncertainty, and observation uncertainty, to construct a probabilistic uncertainty model. Using this model, we can represent the gradual reduction of the uncertainty in motion prediction, which we usually experience in many situations. Based on this probabilistic model, the motion planner repeatedly selects the best motion in a decision-theoretic manner.

Currently we assume that the robot can determine the path of an obstacle at a branching point just after the obstacle passes the point. However, this assumption may not be effective in some cases, especially when the branching point is far from the robot and the observation uncertainty is large. An extension to the current modeling is to consider the observation uncertainty in determining the path of the obstacle.

In this paper, we have described simulation results on a simple case where there is only one moving obstacle. Another future work is to apply the method to multiple-obstacle cases. Although, in principle, the proposed method can be applied to multiple-obstacle cases, several extensions would be necessary such as modeling the interaction between obstacles.

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